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Calculated Hovering Helicopter Flight Dynamics with a Circulation-Controlled Rotor

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The flight dynamics of a hovering helicopter with a circulation-controlled rotor are analyzed. The influence of the rotor blowing coefficient on the calculated eigenvalues of the helicopter motion is examined for a range of values of the rotor lift and the blade flap frequency. The control characteristics of a helicopter with a circulation-controlled rotor are discussed. The principal effect of the blowing is a reduction in the rotor speed stability derivative. Above a critical level of blowing coefficient, which depends on the flap frequency and rotor lift, negative speed stability is produced and the dynamic characteristics of the helicopter are radically altered. The handling qualities of a helicopter with negative speed stability are probably unacceptable without a stability augmentation system.

Introduction

A circulation-controlled rotor uses blowing at the blade trailing edge to control the rotor lift, in place of the geometric pitch control of conventional rotors. The blade section lift is given by the product of the dynamic pressure, chord, and lift coefficient, $L = \frac{1}{2}\rho V^2 c c_l(\alpha, C_\mu)$, where the lift coefficient depends now on the blowing coefficient C_μ as well as on the angle-of-attack. The blowing coefficient is defined as $C_\mu = \dot{m}V_j/(\frac{1}{2}\rho V^2 c)$, where $\dot{m}V_j$ is the jet momentum. With a conventional rotor a perturbation of the blade inplane velocity V influences the lift by changing the section dynamic pressure and angle-of-attack. With a circulation-controlled rotor, inplane velocity perturbations also influence the lift by changing the section blowing coefficient:

$$\delta L = \delta(\tfrac{1}{2}\rho V^2 c c_l) = \tfrac{1}{2}\rho V^2 c[c_{l_\alpha}\delta\alpha + c_{l_\mu}\delta C_\mu + 2c_l(\delta V/V)]$$

where $c_{l_\alpha} = \partial c_l/\partial\alpha$ and $c_{l_\mu} = \partial c_l/\partial C_\mu$. Assuming that the jet momentum is fixed, we have

$$\delta C_\mu = \left(\frac{\dot{m}V_j}{\frac{1}{2}\rho c}\right)\left(-\frac{2\delta V}{V^3}\right) = -2C_\mu \frac{\delta V}{V}$$

The additional lift change due to inplane velocity perturbations with trailing-edge blowing will alter the dynamic characteristics of the circulation-controlled rotor compared to conventional rotors. One concern is the influence of the blowing on the helicopter flight dynamics. This paper presents the results of an analysis of the hovering helicopter flight dynamics with a circulation-controlled rotor.

Equations of Motion

Consider a hovering helicopter with the center of gravity a distance h below the rotor hub, and on the shaft axis. Then the longitudinal/lateral dynamics decouple from the helicopter vertical and yaw motions. The degrees of freedom of the helicopter rigid body motion are the longitudinal velocity u_F, the lateral velocity v_F, the pitch angle θ_F, and the roll angle ϕ_F. For helicopter flight dynamics analyses it is generally sufficient to consider a quasistatic model for the rotor hub reactions in response to shaft motion control, and gusts. With the quasistatic or low-frequency solution, the rotor model does not add degrees of freedom to the system, rather the rotor is represented by a set of stability derivatives. Cyclic blowing control $\Delta C_\mu = \delta_c \cos\psi + \delta_s \sin\psi$ is considered, as well as conventional cyclic pitch control $\Delta\theta = \theta_c \cos\psi + \theta_s \sin\psi$ (ψ is the rotor blade azimuth angle). The rotor couples the helicopter longitudinal and lateral motions, but it is convenient and often sufficiently accurate to decouple the equations and analyze the longitudinal and lateral dynamics separately. In Laplace form, the decoupled longitudinal and lateral equations of motion are[1]:

$$\begin{bmatrix} s - X_u & -X_q s + g \\ -M_u & s^2 - M_q s \end{bmatrix} \begin{pmatrix} u_F \\ \theta_F \end{pmatrix} = \begin{bmatrix} X_\theta & X_\delta \\ M_\theta & M_\delta \end{bmatrix} \begin{pmatrix} \theta_s \\ \delta_s \end{pmatrix}$$

$$\begin{bmatrix} s - Y_v & -Y_p s - g \\ -L_v & s^2 - L_p s \end{bmatrix} \begin{pmatrix} v_F \\ \phi_F \end{pmatrix} = \begin{bmatrix} Y_\theta & Y_\delta \\ L_\theta & L_\delta \end{bmatrix} \begin{pmatrix} \theta_c \\ \delta_c \end{pmatrix}$$

Here s is the Laplace variable and g is the acceleration due to gravity. These equations invert to

$$\begin{pmatrix} u_F \\ \theta_F \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} X_\theta s^2 + (X_q M_\theta - X_\theta M_q)s - M_\theta g \\ M_\theta s + (X_\theta M_u - X_u M_\theta) \end{pmatrix}\theta_s$$
$$\frac{1}{\Delta}\begin{pmatrix} X_\delta s^2 + (X_q M_\delta - X_\delta M_q)s - M_\delta g \\ M_\delta s + (X_\delta M_u - X_u M_\delta) \end{pmatrix}\delta_s$$

where $\Delta = s^3 - (X_u + M_q)s^2 + (X_u M_q - X_q M_u)s + gM_u$; and similarly for the lateral dynamics.

The rotor stability derivatives depend on the characteristics of the blade section lift perturbations. Using for the angle-of-attack $\alpha = \theta - \lambda/V$, where θ is the blade pitch angle and λ is the inflow velocity (positive down through the rotor disk), and $\delta C_\mu = -2C_\mu \delta V/V$ for the blowing coefficient perturbation due to an inplane velocity change, the section lift perturbation

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becomes

$$\delta L = \tfrac{1}{2}\rho V^2 c c_{l_\alpha}(\delta\theta - \delta\lambda/V) + \tfrac{1}{2}\rho V^2 c c_{l_\mu}\delta C_\mu + \tfrac{1}{2}\rho V^2 c[c_{l_\alpha}\lambda/V + 2(c_l - C_\mu c_{l_\mu})](\delta V/V)$$

The lift due to cyclic pitch control ($\delta\theta$) and due to vertical velocity perturbations of the helicopter ($\delta\lambda$) is proportional to the lift curve slope, and hence is relatively unaffected by the trailing-edge blowing for unstalled operating conditions. The lift due to cyclic blowing control (δC_μ) is proportional to the section derivative c_{l_μ}, which depends primarily on the blowing coefficient.[2] The lift change due to inplane velocity perturbations (δV) depends on the blade lift and on the blowing coefficient. For a constant trim lift coefficient (which also fixes the induced velocity λ), the derivative $\partial L/\partial V$ is reduced as C_μ increases, and can even be negative for a large enough blowing coefficient. Approximating the airfoil characteristics by $c_l = a\alpha + bC_\mu^p$, the inplane velocity perturbation term can also be written as

$$\delta L = \tfrac{1}{2}\rho V^2 c[a\lambda/V + 2a\alpha + 2bC_\mu^p(1-p)](\delta V/V)$$
$$= \tfrac{1}{2}\rho V^2 c[2a\theta - a\lambda/V + 2bC_\mu^p(1-p)](\delta V/V)$$

to show the dependence on angle-of-attack and collective pitch. Since circulation-controlled airfoils show a lift dependence on blowing such that $p \leq 1$ (see Ref. 2), it follows that negative $\partial L/\partial V$ must be associated with operating conditions at negative angle-of-attack, hence high blowing for a given lift.

The handling qualities of the hovering helicopter longitudinal dynamics are determined primarily by the pitch damping derivative M_q and the speed stability derivative M_u (similarly the lateral dynamics depend primarily on the roll damping L_p and the speed stability or dihedral effect L_v). The pitch and roll damping produced by the rotor are due to the time lag of the tip-path plane tilt relative to the shaft during angular motions of the helicopter. This tip-path plane tilt produces the aerodynamic moment necessary to precess the rotor disk to follow the shaft motion. The source of the rotor aerodynamic moment is the flap moment due to flapping velocity, in other words, a blade lift change due to an angle-of-attack perturbation. It follows from the expression for $\partial L/\partial V$ in the preceding paragraph that M_q and L_p are not influenced greatly by the trailing-edge blowing. (There is a minor effect since pitching or rolling about the helicopter center of gravity will produce an inplane hub velocity.) The speed stability produced by the rotor is, however, due to the flap moments and hub forces which occur during inplane motions of the rotor hub. Therefore, M_u and L_v are directly influenced by the trailing-edge blowing of the circulation-controlled rotor. With zero or low blowing, a forward velocity of the helicopter increases the blade lift on the advancing side and decreases it on the retreating side of the disk. The rotor responds with a rearward tilt of the tip-path plane, which produces a thrust-vector tilt and a hub moment. The resulting pitchup moment on the helicopter is the speed stability derivative ($M_u > 0$). At constant lift, increased blowing coefficient reduces $\partial L/\partial V$ of the blade section. For sufficiently high blowing coefficient the lift can actually decrease due to an inplane velocity increase, resulting in negative speed stability. In summary, the principal influence of blowing on the rotor stability derivatives is a change in the speed stability M_u and L_v, which are reduced for $C_\mu > 0$. For sufficiently high blowing, negative speed stability is possible ($M_u > 0$ and $L_v < 0$ correspond to positive speed stability). An expression for the speed stability derivative is given in the Appendix.

The characteristic equation of the helicopter longitudinal dynamics is

$$s^3 - (X_u + M_q)s^2 + (X_u M_q - X_q M_u)s + gM_u = 0$$

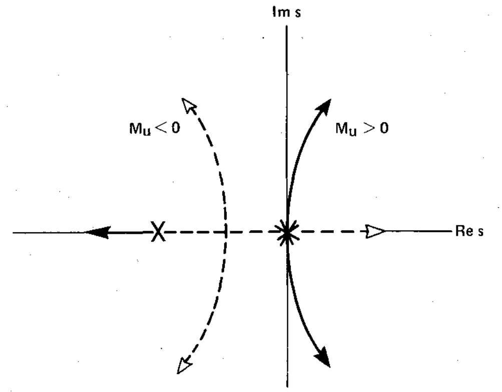

Fig. 1 Influence of speed stability M_u on the roots of the hovering helicopter longitudinal dynamics.

The influence of the speed stability M_u on the three roots of this equation is shown in Fig. 1. With no blowing the helicopter has positive speed stability, $M_u > 0$. Then the roots of the hover longitudinal dynamics consist of a stable real root due to the pitch damping M_q, and a mildly unstable long period oscillatory mode due to the speed stability. With a low level of blowing, the speed stability derivative M_u is reduced. The result is a reduction in the damping of the real root and an increase in the period and time to double-amplitude of the oscillatory mode. This change in the oscillatory mode is favorable and the influence on the real root will be small with a hingeless rotor. With sufficiently high blowing the rotor will have negative speed stability, $M_u < 0$. Then the roots of the hover dynamics consist of two stable real roots (or a stable oscillatory mode), and an unstable real divergence due to the speed stability. The time to double-amplitude of this unstable mode can be unacceptably short. There are similar effects of the blowing on the hovering helicopter lateral dynamics.

Calculated Dynamic Characteristics

The calculated flight dynamics of the hovering helicopter with a circulation-controlled rotor will be presented for the following example: rotor solidity $\sigma = 0.085$; blade Lock number $\gamma = 5$; mast height $h/R = 0.3$; precone $\beta_p = 3$ deg; pitch radius of gyration $(k_y/R)^2 = 0.10$; and a gravitational constant of $g/\Omega^2 R = 0.002$. A rotor speed of $\Omega = 27$ rad/s is used to obtain dimensional results (hence with $g/\Omega^2 R = 0.002$, the rotor tip speed and radius are $\Omega R = 180$ m/s and $R = 6.7$ m). The circulation-controlled airfoil characteristics at constant Mach number were approximated by $c_l = a\alpha + bC_\mu^p$ with $a = 7.0$, $b = 10.8$, and $p = 2/3$ (based on the data of Ref. 2). For the blade section drag coefficient, $c_d = 0.012$ was used. The hover-induced velocity was obtained using the momentum theory result $\lambda_0 = \kappa_h\sqrt{C_T/2}$, with the empirical factor $\kappa_h = 1.15$. It was assumed that the blade section blowing coefficient varied inversely with the radial station along the span: $C_\mu = C_{\mu_t}(R/r)$. The flight dynamics are considered over a range of rotor lift (thrust coefficient to solidity ratios of $C_T/\sigma = 0.05$ to 0.15), as a function of the tip-blowing coefficient C_{μ_t}. Several values of the flap frequency are considered: $\nu = 1$, as for a teetering or gimballed rotor; $\nu = 1.04$, representative of an offset-hinge articulated rotor; and $\nu = 1.09$ and 1.18, representative of hingeless rotors.

Figures 2 and 3 show the variation of the speed stability derivative M_u with the tip blowing coefficient C_{μ_t} and the rotor lift C_T/σ, for flap frequencies of 1.0 and 1.09/rev, respectively. The value of the blowing coefficient at which the speed stability becomes zero is observed to increase with C_T/σ and ν. The rotor in hover is axisymmetric, so the lateral speed stability derivative L_v differs from the longitudinal derivative M_u only because of the difference between the roll and pitch moments of inertia of the helicopter. Hence both of these derivatives become zero at the same value of C_{μ_t}. With a

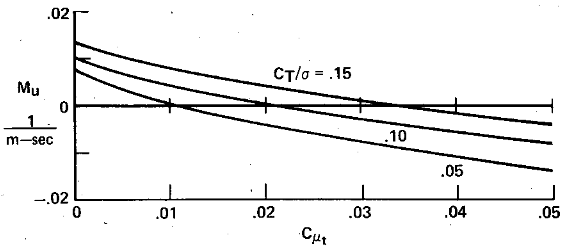

Fig. 2 Speed stability derivative M_u as a function of C_T/σ and C_{μ_t}, for a flap frequency $\nu = 1.0$ (the effect of the lift deficiency function is negligible).

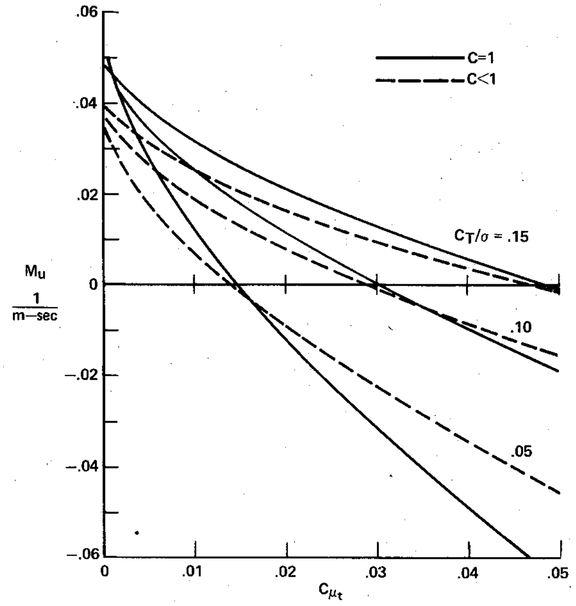

Fig. 3 Speed stability derivative M_u as a function of C_T/σ and C_{μ_t}, for a flap frequency $\nu = 1.09$; without unsteady wake effects ($C = 1$) and with the lift deficiency function ($C < 1$).

quasistatic rotor model, the effects of unsteady aerodynamics can be accounted for by using a lift deficiency function C (defined in the Appendix). With a flap frequency above 1/rev, the reduction of the rotor aerodynamic hub moments in hover due to the unsteady wake effects can be large, as shown in Fig. 3. Note, however, that the lift deficiency function has little influence on the point where the speed stability becomes zero. Hence the calculated roots presented below were obtained without the unsteady wake effects ($C = 1$). The influence of the lift deficiency function ($C < 1$) on the flight dynamics was examined in detail in Ref. 3.

The decoupled equations for the hovering helicopter longitudinal and lateral flight dynamics are of lower order than the coupled equations, hence are easier to solve and easier to interpret. The coupling of the longitudinal and lateral motions is particularly strong with a hingeless rotor however. The solution for the coupled dynamics of a helicopter with a circulation-controlled rotor was compared in Ref. 3 with the results of separate solutions of the decoupled equations for the longitudinal and lateral dynamics. It was found that the principal influence of the coupling is on the eigenvectors; the eigenvalues are well estimated by the decoupled equations. In particular, the behavior of the roots at high values of the blowing coefficient are correctly obtained. Also, the influence of the blowing coefficient on the longitudinal dynamics and on the lateral dynamics is very similar. Hence only the longitudinal roots, obtained from the decoupled equations, are examined here.

Figures 4 to 6 show the influence of rotor trailing-edge blowing on the calculated longitudinal flight dynamics in

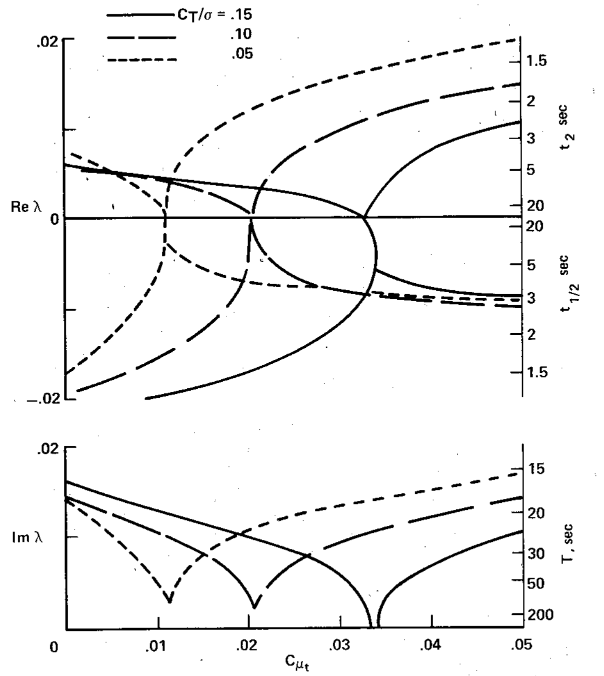

Fig. 4 Calculated roots of the hover longitudinal dynamics as a function of the tip blowing coefficient, for flap frequency $\nu = 1.0$.

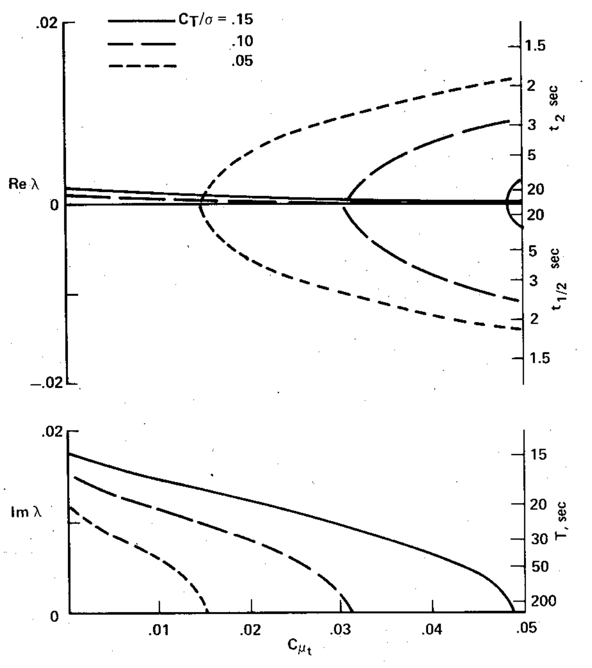

Fig. 5 Calculated roots of the hover longitudinal dynamics as a function of the tip blowing coefficient, for flap frequency $\nu = 1.09$.

hover. The real and imaginary parts of the roots are plotted as a function of the tip-blowing coefficient C_{μ_t}; the right-hand axes show the corresponding time to double- or half-amplitude and period of the mode. Figures 4 and 5 give the roots for the articulated rotor ($\nu = 1$) and the hingeless rotor ($\nu = 1.09$), respectively, at several values of the rotor lift. Figure 6 gives the roots at $C_T/\sigma = 0.10$ for several values of the flap frequency. The variation of the roots as the blowing coefficient increases follows from the influence of blowing on the speed stability (Figs. 2 and 3) and the influence of speed stability on the roots (Fig. 1). Consider Fig. 4; at zero blowing the speed stability is positive, hence there is a single stable root (with negative real part in Fig. 4) and a pair of oscillatory unstable roots (with positive real part and nonzero imaginary

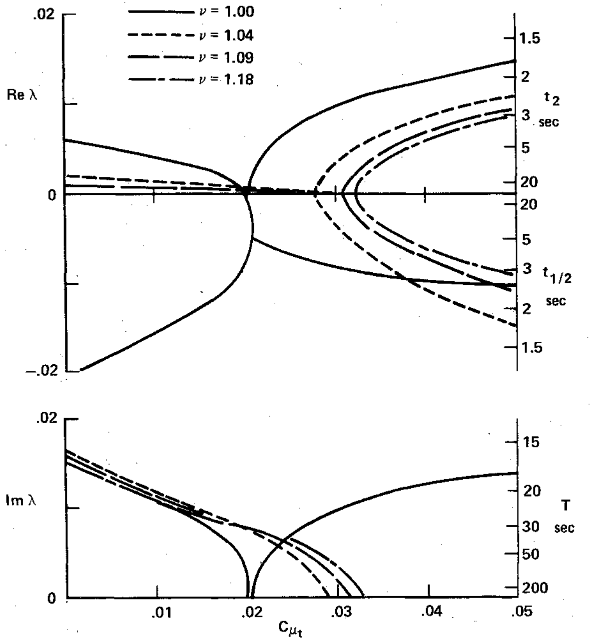

Fig. 6 Calculated roots of the hover longitudinal dynamics as a function of the tip blowing coefficient, at a rotor lift $C_T/\sigma = 0.10$.

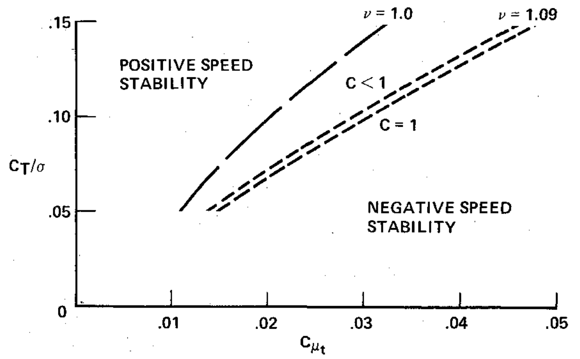

Fig. 7 Boundary for zero speed stability.

part in Fig. 4). As C_{μ_t} increases and the speed stability decreases, the damping of the stable real root decreases while the period and time to double-amplitude of the unstable oscillatory mode increase. At a value of C_{μ_t} which depends on the rotor lift, the two complex roots reach the origin, and for still higher blowing become two real roots, one stable and one unstable (see Fig. 1). For this case with flap frequency $\nu = 1.0$, the two stable real roots then meet on the real axis and break off to form a stable oscillatory mode. With a hingeless rotor (Fig. 5) the behavior is similar, except that the stable real root due to the pitch damping has much larger magnitude, and thus does not combine with the other stable real root at high C_{μ_t}. Indeed, this pitch root is off the scale of Figs. 5 and 6, even for the offset-hinge articulated rotor ($\nu = 1.04$). The influence of the blowing coefficient on the roots is similar for the lateral dynamics.

The point at which the complex roots reach the origin, and form a pair of real roots, corresponds to zero speed stability ($M_u = 0$ or $L_v = 0$). Note that the unstable real roots initially increase very quickly after this critical blowing coefficient is reached. Therefore, the blowing coefficient cannot be increased much above this point before the time to double-amplitude becomes unacceptably small. Figure 7 shows the zero speed stability boundary as a function of C_T/σ and C_{μ_t} for the numerical example considered here. For a high enough blowing coefficient, or a small enough rotor lift, the helicopter will have negative speed stability and hence an unstable real root for the hover dynamics.

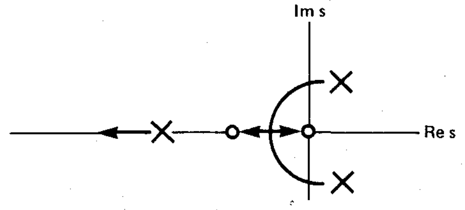

Fig. 8 Influence of rate plus proportional feedback of pitch attitude on the helicopter longitudinal dynamics, for positive speed stability: ×, open loop poles; ○, open loop zeros.

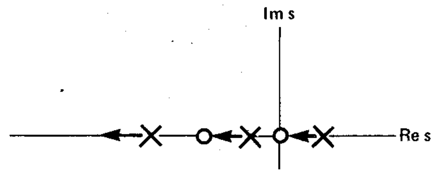

Fig. 9 Influence of rate plus proportional feedback of pitch attitude on the helicopter longitudinal dynamics, for negative speed stability: ×, open loop poles; ○, open loop zeros.

Control Characteristics

The response of the helicopter longitudinal velocity u_F to cyclic pitch or blowing control has two zeros, a complex conjugate pair of relatively large magnitude. The pitch angle θ_F has a single real zero of very small magnitude. The zeros of the lateral response to control are similar. The zeros of the response to cyclic blowing δ_s are identical to the zeros of the response to cyclic pitch θ_s if $(X_\delta M_\theta)/(X_\theta M_\delta) = 1$, which is exactly satisfied with $\nu = 1.0$, and is satisfied within 10% for flap frequencies above 1/rev. Calculations of the zeros for the numerical example considered in the preceding section confirm that the zeros for cyclic blowing and cyclic pitch are essentially identical. Hence it is only the pole configuration which influences the helicopter response to control.

A conventional helicopter ($C_{\mu_t} = 0$) has a mildly unstable long period oscillation in hover, but the motion can be stabilized by the pilot or the automatic flight control system using rate plus proportional feedback of the pitch motion ($\theta_s = -K(\tau s + 1)\theta_F$, and similarly for the lateral dynamics). The root locus for such feedback is shown in Fig. 8 (the θ_F/θ_s response has a single zero at the origin, and the lead adds another at $s = \tau^{-1}$). A moderate level of blowing will improve the helicopter control characteristics by reducing the speed stability; however, blowing large enough to produce negative speed stability radically alters the pole configuration. In this case, as shown in Fig. 9, rate plus proportional feedback of the pitch attitude ($\delta_s = -K(\tau s + 1)\theta_F$) improves the dynamic characteristics by increasing the time to double amplitude of the real divergence, but it does not completely stablize the system regardless of the value of the lead τ. To obtain a stable system, feedback of the velocity perturbation could be added to counter the negative speed stability, and then rate plus proportional feedback of pitch used [$\delta_s = -K_\theta(\tau s + 1)\theta_F + K_u u_F$]. Alternatively, integral feedback of pitch could be used, as well, to produce a stable system [$\delta_s = -K(\tau_R s + 1 + [\tau_I s]^{-1})\theta_F$]. Such control laws would likely be too complex for the pilot to comfortably handle. In any case, an automatic stability augmentation system is clearly desirable with negative speed stability. Data in Ref. 4 regarding the effects of speed stability indicate that the handling qualities are probably unacceptable if M_u is less than about -0.01 m-s^{-1}, and clearly it is desirable to have positive speed stability ($M_u \geq 0$).

Concluding Remarks

With a conventional rotor, an increase in the blade inplane velocity increases the dynamic pressure and decreases the induced angle-of-attack of the section, thereby increasing the lift. With a circulation-controlled rotor, an inplane velocity increase reduces the blowing coefficient, and thus tends to reduce the lift. If the blowing is high enough that there is a net reduction in the blade lift for an inplane velocity increase, the dynamic behavior of the rotor is radically altered. For the hovering flight dynamics, there is a critical blowing level, above which the rotor produces a negative speed stability derivative. The handling qualities of a helicopter with negative speed stability are probably unacceptable without a stability augmentation system.

The present analysis has considered only the case of hovering flight. At low to moderate forward speeds the handling qualities are still dominated by the pitch/roll damping and speed stability, hence should retain characteristics similar to those found in hover; there are even additional possible sources of negative speed stability, particularly aerodynamic interference between the rotor and airframe. At high forward speeds the helicopter longitudinal handling qualities are strongly influenced by the angle-of-attack stability derivative (M_w, likely to be unstable for a hingeless rotor helicopter even with a horizontal tail). A consideration of the influence of this derivative on the eigenvalues suggests that negative speed stability, while no longer playing a dominant role, would still retain a significant unfavorable influence on the handling qualities in high-speed forward flight.

Appendix: Hovering Helicopter Speed Stability Derivative

The equations of motion for the helicopter rigid body degrees of freedom, rotor flap motion, hub reactions, and inflow perturbation were obtained from the aeroelastic analysis of Ref. 5. For the rotor, only the flap motion was considered, specifically the tip-path plane tilt degrees of freedom. For the flap mode shape, rigid body rotation with no hinge offset was assumed in order to simplify the aerodynamic and inertial coefficients. Hingeless rotors were modeled by retaining an arbitrary flap frequency. Perturbations of the rotor-induced velocity, linearly varying over the rotor disk, were included to account for the rotor unsteady aerodynamics. The influence of these inflow perturbations principally takes the form of a lift deficiency function C multiplying the aerodynamic coefficients. A quasistatic solution for the rotor flap response was used. The flapping velocity and acceleration terms were dropped and then the flap equations were solved for the tip-path plane tilt response. The flapping velocity terms were dropped from the hub reactions and the solutions for the tip-path plane tilt substituted. From the resulting expansion of the rotor low-frequency response in terms of the perturbation body motion, the stability derivatives for the flight dynamics analysis were identified. The result for the speed stability derivative M_u is as follows:

$$M_u = \frac{g}{k_y^2\Omega}\,\frac{1}{2C_T/\sigma a}\left\{\frac{\nu^2-1}{\gamma/8}\,\frac{1}{1+N_*^2}M_\mu - \frac{h}{R}\,(2\lambda_0 M_\mu - H_\mu)\right.$$

$$+\frac{h}{R}\,\frac{8M_\mu}{1+N_*^2}\left[\left(2+(1-C)N_*\frac{\nu^2-1}{C\gamma/8}\right)\frac{C_T}{\sigma a}\right.$$

$$\left.\left.+\frac{\nu^2-1}{\gamma/8}\left(N_*\frac{\lambda_0}{4}+\frac{\beta_0}{6}\right)\right]\right\}$$

where

$$N_* = \frac{\nu^2-1}{\gamma/8} + K_P$$

and the lift deficiency function (from Ref. 5) is

$$C = \frac{1}{1+(\sigma a \kappa_h^2)/(8\lambda_0)}$$

The lateral speed stability is $L_v = -(k_y^2/k_x^2)M_u$. The parameters required in these expressions are

g = acceleration due to gravity
k_y = pitch radius of gyration
k_x = roll radius of gyration
Ω = rotor angular velocity
C_T = rotor thrust coefficient
σ = rotor solidity ratio
a = blade section lift-curve slope
ν = flap frequency (per rev, in the rotating frame)
γ = blade Lock number
h = rotor mast height
R = rotor radius
λ_0 = induced velocity
β_0 = blade coning angle

The momentum theory value for the induced velocity in hover is $\lambda_0 = \kappa_h\sqrt{C_T/2}$, where κ_h is an empirical constant.

The rotor aerodynamic coefficients M_μ and H_μ are, respectively, the flap moment and blade drag force due to inplane velocity of the hub (see Ref. 5). It is only through these aerodynamic coefficients that the trailing-edge blowing influences the flight dynamics. To evaluate M_μ and H_μ, it was assumed that the blade has a constant chord and linear twist (θ_{tw}), and that the blowing coefficient varies along the blade span as $C_\mu = C_{\mu_t}(R/r)$. The circulation-controlled airfoil characteristics were approximated by $c_l = a\alpha + bC_\mu^p$. The results for M_μ and H_μ are

$$M_\mu = \frac{2C_T}{\sigma a} + \frac{\lambda_0}{4} - C_{\mu_t}^p\,\frac{pb}{a(3-p)}$$

$$H_\mu = \lambda_0\left(\frac{3C_T}{\sigma a} - \frac{\theta_{tw}}{8} + \frac{3}{4}\lambda_0\right) + \frac{3c_d}{4a} + \frac{\lambda_0 b}{2a}C_{\mu_t}^p\left(\frac{1-2p}{1-p} - \frac{3}{3-p}\right)$$

and for the blade coning and collective pitch angles:

$$\beta_0 = \frac{\nu^2-1}{\nu^2}\beta_p + \frac{\gamma}{\nu^2}\left(\frac{3}{4}\frac{C_T}{\sigma a} + \frac{\theta_{tw}}{160} + \frac{\lambda_0}{48}\right.$$

$$\left.-C_{\mu_t}^p\,\frac{bp}{a3(3-p)(4-p)}\right)$$

$$\theta_{75} = \frac{6C_T}{\sigma a} + \frac{3}{2}\lambda_0 - C_{\mu_t}^p\,\frac{3b}{a(3-p)}$$

where β_p is the hub precone angle.

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